

# Relativistic Shock Waves and Mach Cones in Viscous Gluon Matter

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## Abstract.

To investigate the formation and the propagation of relativistic shock waves in viscous gluon matter we solve the relativistic Riemann problem using a microscopic parton cascade. We demonstrate the transition from ideal to viscous shock waves by varying the shear viscosity to entropy density ratio  $\eta/s$ . Furthermore we compare our results with those obtained by solving the relativistic causal dissipative fluid equations of Israel and Stewart (IS), in order to show the validity of the IS hydrodynamics. Employing the parton cascade we also investigate the formation of Mach shocks induced by a high-energy gluon traversing viscous gluon matter. For  $\eta/s = 0.08$  a Mach cone structure is observed, whereas the signal smears out for  $\eta/s \geq 0.32$ .

## 1. Introduction

Jet quenching has been discovered in heavy-ion collisions at BNL's Relativistic Heavy Ion Collider (RHIC). In this context, very exciting jet-associated particle correlations [2] have been observed, which indicates the formation of shock waves in the form of Mach cones [3], induced by supersonic partons moving through the quark-gluon plasma (QGP). Measuring the Mach cone angle could give us the possibility to extract the equation of state of the QGP.

Shock waves can only develop in a medium which behaves like a fluid. The large elliptic flow coefficient  $v_2$  measured at RHIC [4] implies that the QGP created could be a nearly perfect fluid with a small viscosity. Calculations of viscous hydrodynamics [5] and microscopic transport theory [6, 7] have estimated the shear viscosity to the entropy density ratio  $\eta/s$  to be less than 0.4. There is still an open question if this upper limit of the  $\eta/s$  ratio is small enough to allow the formation of Mach shocks.

In this work we address the question, whether and when relativistic shock waves and Mach cones can develop in viscous gluon matter for given  $\eta/s$  values. For this purpose we consider first the relativistic Riemann problem [8], which we solve within the kinetic theory and the Israel-Stewart (IS) theory of viscous hydrodynamics for comparisons. Here we employ the microscopic parton cascade BAMPS (Boltzmann Approach of MultiParton Scatterings) [9] and a solver of the

IS equations, vSHASTA (viscous SHArp and Smooth Transport Algorithm) [10]. Particularly, we demonstrate agreements between the two approaches for matter with (extreme) small  $\eta/s$  values and also show deviations when the  $\eta/s$  ratio becomes large, which implies the invalidity of the IS theory. Second, we consider a traverse of a high-energy gluon through gluon matter and investigate the formation of shocks in form of Mach cones. Preliminary results are obtained by using BAMPS.

## 2. BAMPS and vSHASTA

BAMPS is a microscopic transport model solving the Boltzmann equation

$$p^\mu \partial_\mu f(x, p) = C(x, p) \quad (1)$$

for on-shell particles with the collision integral  $C(x, p)$ . The algorithm for collisions is based on the stochastic interpretation of the transition rate [9]. In this study, we consider only binary gluon scattering processes with an isotropic cross section, which is adjusted locally at each time step to keep a constant  $\eta/s$  value [11, 12, 13].

Simulations of particle evolution in space and time are performed in a static box, where the whole box is divided into spatial cells with a volume  $V_{\text{cell}} = \Delta x \Delta y \Delta z$ . Collisions of particles in same cells are simulated by Monte Carlo technique according to the individual collision probability within a time step of  $\Delta t$ ,

$$P_{22} = v_{\text{rel}} \frac{\sigma}{N_{\text{test}}} \frac{\Delta t}{V_{\text{cell}}}, \quad (2)$$

where  $\sigma$  is the total cross section,  $v_{\text{rel}} = (p_1 + p_2)^2 / (2E_1 E_2)$  denotes the relative velocity of the two incoming particles with four momenta  $p_1, p_2$  and  $N_{\text{test}}$  is the testparticle number. The testparticle method is introduced to reduce statistical fluctuations and is implemented such that the mean free path is left invariant.

The relationship between the shear viscosity  $\eta$  and the total cross section  $\sigma$  is given by  $\eta = 4e/(15R^{tr})$  [12], where  $R^{tr} = n\langle v_{\text{rel}}\sigma^{tr} \rangle = 2n\langle v_{\text{rel}}\sigma \rangle/3$  is the transport collision rate in case of isotropic scattering processes.  $\langle \rangle$  stands for ensemble average in local rest frame. We obtain

$$\eta = \frac{2}{3} e \lambda_{\text{mfp}} \quad (3)$$

for an ultrarelativistic massless gas. Here the LRF energy density is  $e = 3nT$ ,  $n$  is the LRF particle density,  $T$  the temperature and  $\lambda_{\text{mfp}} = 1/(n\langle v_{\text{rel}}\sigma \rangle)$  is the particle mean free path. In kinetic equilibrium the LRF entropy density is given by  $s = 4n - n \ln \lambda$ , where  $\lambda = n/n_0$  is the fugacity, the ratio of the LRF number density to the one in thermal equilibrium  $n_0 = 16T^3/\pi^2$ .

vSHASTA solves the IS equations of dissipative hydrodynamics for shear pressure and heat flow. In 1+1 dimensions the relaxation equations for heat conductivity and shear stress are

$$Dq^z = \frac{1}{\tau_q} (q_{NS}^z - q^z) - I_{q1}^z - I_{q2}^z - I_{q3}^z, \quad (4)$$

$$D\pi = \frac{1}{\tau_\pi} (\pi_{NS} - \pi) - I_{\pi1} - I_{\pi2} - I_{\pi3}, \quad (5)$$

with

$$I_{q1}^z = \frac{1}{2} q^z \left( \theta_z + D \ln \frac{\beta_1}{T} \right), \quad (6)$$

$$I_{q2}^z = -q^z v_z \gamma_z^3 (\partial_t v_z + v_z \partial_z v_z), \quad (7)$$

$$I_{q3}^z = \frac{1}{5} [\gamma_z^2 (v_z \partial_t \pi + \partial_z \pi) + \gamma_z \pi (v_z \theta_z + \gamma_z \partial_t v_z)] \quad (8)$$

and

$$I_{\pi 1} = \frac{1}{2}\pi \left( \theta_z + D \ln \frac{\beta_2}{T} \right), \quad (9)$$

$$I_{\pi 2} = \frac{10}{9}(q^z \gamma_z^2) (\partial_t v_z + v_z \partial_z v_z), \quad (10)$$

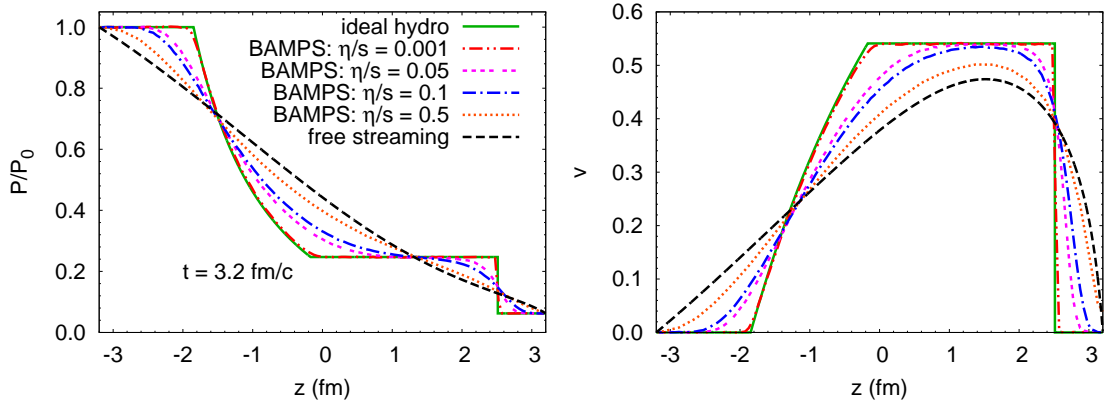
$$I_{\pi 3} = \frac{2}{9} \left( v_z \partial_t q^z + \partial_z q^z - \frac{q^z v_z}{\gamma_z} \theta_z \right), \quad (11)$$

where  $q_{NS}^z$  and  $\pi_{NS}$  are the Navier-Stokes values [10] and  $D \equiv u^\mu \partial_\mu$ .  $\tau_q$  and  $\tau_\pi$  are the relaxation times, respectively. For vanishing shear pressure and heat conductivity the IS equations reduce to the equations of ideal hydrodynamics.

### 3. The relativistic Riemann problem

The relativistic Riemann problem [8] is a well-known shock problem in ideal hydrodynamics. Initially matter is separated by a membrane at  $z = 0$  in two regions,  $z < 0$  and  $z > 0$ , with two different pressures  $P_0$  and  $P_4$ . The velocities on both sides are  $v_0 = v_4 = 0$ . The matter is assumed to be homogeneous in transverse direction.

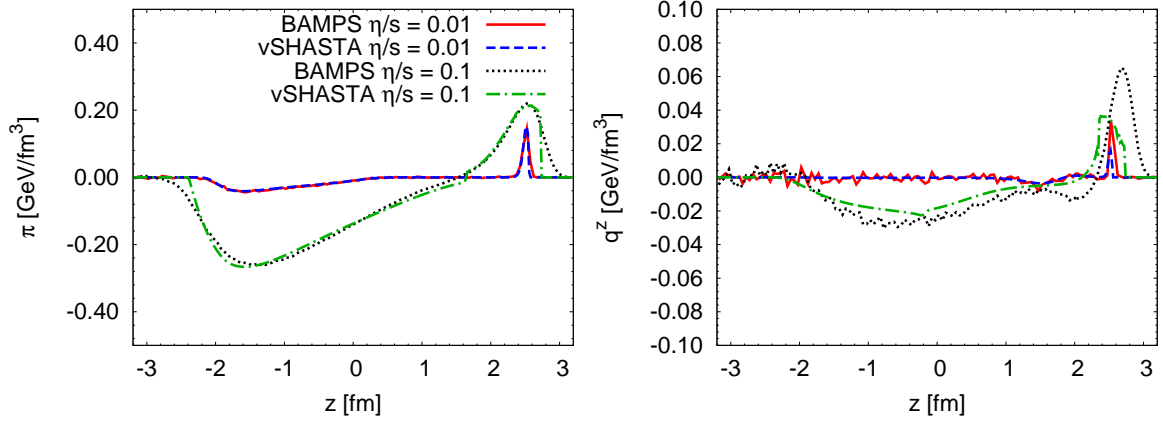
Removing the membrane in the case for a perfect fluid, i.e.,  $\eta/s = 0$ , we observe (the green curve in Fig. 1) a propagation of a shock wave to the right with a larger velocity than the speed of sound and a rarefaction wave to the left exactly with the speed of sound.



**Figure 1.** Spatial profile of pressure (left panel, normalized by the initial value) and collective velocity (right panel) at  $t = 3.2 \text{ fm}/c$ .

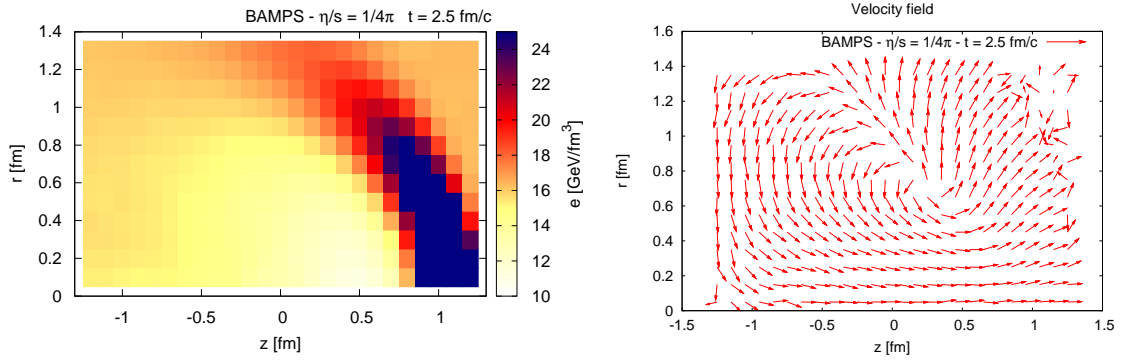
The BAMPS solutions for various  $\eta/s$  are depicted in Fig. 1. In particular, the BAMPS result for  $\eta/s = 0.001$  reproduces the ideal solution for a perfect fluid to a very high precision. With the increasing  $\eta/s$  value we see a clear transition from the formation of shock waves in ideal fluid to the smearing out in free streaming of particles [11]. The characteristic shock profiles disappear for large  $\eta/s$  values.

In Fig. 2 we show comparisons between BAMPS and vSHASTA for the shear pressure  $\pi = \pi^{zz}/\gamma^2$  and heat flow  $q^z = h\gamma^2(vN^0 - N^3)$  at  $t = 3.2 \text{ fm}/c$ .  $\pi^{\mu\nu}$  is the shear stress tensor,  $N^\mu$  is the particle four-flow,  $\gamma^2 = (1 - v^2)^{-1}$  and  $h = (e + P)/n$  is the enthalpy per particle. For  $\eta/s = 0.01$  we see a very good agreement between vSHASTA and BAMPS, whereas for  $\eta/s = 0.1$  deviations in the region of the shock front and rarefaction fan appear. In the case of  $\eta/s = 0.1$ , the local system at the shock front and rarefaction fan is strongly out of equilibrium and, thus, the applicability of the IS theory is questionable. The microscopic transport approach does not suffer from this drawback.



**Figure 2.** Comparison of BAMPS and vSHASTA for the shear pressure  $\pi$  and heat flow  $q^z$ .

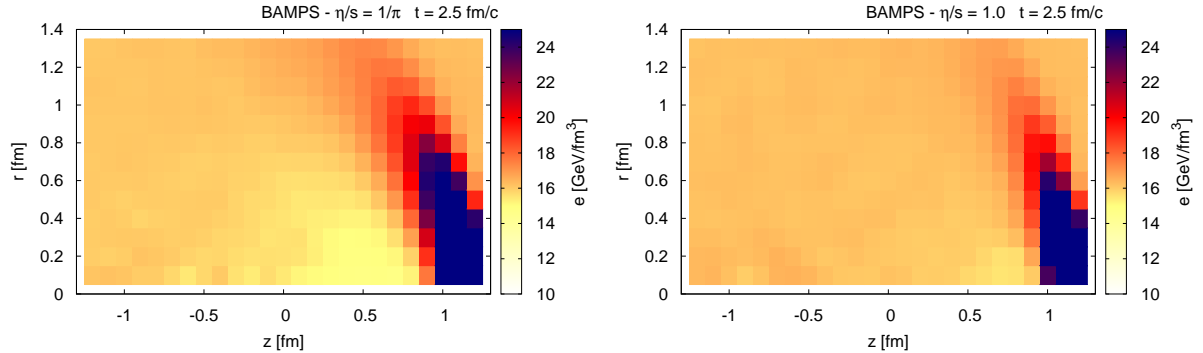
#### 4. Mach Cones in BAMPS



**Figure 3.** Spatial profile of the energy density (left panel) and the velocity field (right panel). The  $\eta/s$  value of the thermal medium is  $1/4\pi$ . The initial velocity of the high-energy gluon is in  $z$ -direction. The length of the arrays in the right panel is unit, i.e., only the direction of the velocity is shown.

Formation of 3-dimensional shock waves in form of Mach cones is investigated by shooting a gluon with energy of 20 GeV into a thermal gluon medium with a temperature of  $T = 400 \text{ MeV}$ . The thermal medium is embedded in a static box.

Fig. 3 shows spatial profiles of the energy density and velocity field at  $t = 2.5 \text{ fm}/c$  for a medium with  $\eta/s = 1/4\pi$ . The energy, which the gluon probe lost due to interactions with the medium, creates a shock wave that propagates in form of a Mach cone. The energy density of the region behind the Mach cone is smaller than the initial energy density of the medium. This region is called a diffusion wake. Collective behavior of the medium response is also clearly seen in the profile of the velocity field. Our results agree qualitatively with those found in [14, 15]. For higher values of the  $\eta/s$  ratio the typical Mach cone structure smears out as observed in Fig. 4. The strength of the Mach cone signal and also the lower energy density region behind the shock front become weaker because of weaker particle interactions in medium with larger  $\eta/s$ .



**Figure 4.** Same as Fig. 3, but for  $\eta/s = 1/\pi$  (left panel) and 1.0 (right panel).

## 5. Summary

We have solved the relativistic Riemann problem using BAMPS and vSHASTA, in order to investigate the formation of shock waves in viscous matter. A transition from a characteristic shock profile in ideal fluid to a complete smearing out in particle free streaming has been observed when increasing the  $\eta/s$  value. For high  $\eta/s$  values deviations between BAMPS and vSHASTA occur, which indicates the break down of the IS hydrodynamics. We have also investigated the formation of Mach cones within BAMPS. For  $\eta/s = 1/4\pi$  a Mach cone is clearly visible. For larger values of  $\eta/s = 0.32$  and 1.0 the dissipative effect is so strong that the typical Mach cone structure disappears.

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